= Josephson Diode Effect: Finite-Momentum Pairing and Rashba Spin-Orbit Coupling (Technical Review)

**Abstract:**  
The **Josephson diode effect** (JDE) refers to a nonreciprocal supercurrent in a Josephson junction, where the critical current differs for opposite current directions. This review provides a comprehensive analysis of JDE theories, focusing on two key mechanisms: **finite-momentum Cooper pairing (FMCP)** and **Rashba-like spin–orbit coupling (SOC)** contributions. We begin by revisiting the Josephson effect and the symmetry-breaking requirements for diode behavior. We then examine the theoretical foundations of FMCP, wherein Cooper pairs acquire a finite center-of-mass momentum $q$ in unconventional superconducting states. We present the mathematical framework for finite-$q$ pairing and discuss experimental evidence linking this phenomenon to large diode efficiencies. Next, we analyze how Rashba-type SOC breaks inversion symmetry and, in tandem with time-reversal symmetry breaking, generates asymmetric supercurrent transport. We quantify the influence of SOC strength and interface materials on the diode effect, providing examples of SOC parameters and their impact on diode efficiency. A comparative table summarizes various theoretical approaches (including FMCP-based, anomalous current–phase relations, and vortex/extrinsic mechanisms), highlighting their key features and predictions. Figure placeholders illustrate (i) band structure schematics with spin-split Fermi surfaces, (ii) asymmetric current–phase relations with $\varphi\_{0}$ shifts, and (iii) typical device geometries realizing the Josephson diode effect. We conclude with a discussion of the current understanding, open questions—such as the pursuit of higher efficiency and zero-field diode operation—and the outlook for future research in superconducting electronics and quantum devices.

**Keywords:** Superconducting diode effect; Josephson junction; finite-momentum pairing; Rashba spin–orbit coupling; nonreciprocal supercurrent; symmetry breaking; $\varphi\_{0}$-junction

## 1. Introduction

The Josephson effect in a conventional tunnel junction is characterized by a supercurrent $I$ that depends sinusoidally on the phase difference $\phi$ between two superconductors: $I(\phi) = I\_{c}\sin\phi$. In such symmetric junctions, the critical current $I\_{c}$ is identical for positive (forward) and negative (reverse) bias, i.e. $I\_{c+} = I\_{c-}$. This symmetry is guaranteed by time-reversal and inversion symmetries in the absence of any explicitly symmetry-breaking fields or structures. In contrast, a **Josephson diode** behaves analogously to a semiconductor diode, allowing dissipationless supercurrent preferentially in one direction. Realizing a **superconducting diode effect (SDE)** requires breaking both inversion symmetry and time-reversal symmetry (TRS) in the junction[[1]](https://arxiv.org/html/2502.11717v1#:~:text=the%20Onsager%20reciprocity%20relations%2C%20this,that%20in%20the%20normal%20state)[[2]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=nonreciprocity,causes%20superconductor%20to%20normal%20metal). Under these conditions, the critical currents in the two directions, $I\_{c+}$ and $I\_{c-}$, become unequal ($|I\_{c+}| \neq |I\_{c-}|$), yielding a nonreciprocal current–phase relation and directional superconductivity.

The concept of a superconducting diode was first proposed by Hu *et al.* in 2007, who considered a Josephson junction composed of $p$-type and $n$-type superconductors near a Mott-insulator transition[[3]](https://arxiv.org/html/2502.11717v1#:~:text=The%20concept%20of%20superconducting%20diode,observation%20of%20SDEs%20across%20various)[[4]](https://wucj.physics.ucsd.edu/research/supcond/diode.html#:~:text=Congjun%20Wu%27s%20homepage%20Jiang,Last%20modified). This theoretical work suggested that a polar discontinuity at the interface could produce unidirectional supercurrent flow by breaking inversion symmetry in the superconducting state[[3]](https://arxiv.org/html/2502.11717v1#:~:text=The%20concept%20of%20superconducting%20diode,observation%20of%20SDEs%20across%20various). However, it was not until 2020 that the effect was experimentally observed by Ando *et al.*[[5]](https://www.nature.com/articles/s41467-025-55880-4#:~:text=High,Jiang)[[6]](https://arxiv.org/html/2502.11717v1#:~:text=this%20early%20proposal%2C%20SDEs%20were,30%20%2C%20%2048). They demonstrated a **field-driven SDE** in an artificial superlattice of Nb/V/Ta where a built-in structural inversion asymmetry and an applied magnetic field combined to produce rectified supercurrent flow[[7]](https://arxiv.org/html/2502.11717v1#:~:text=interface%2C%20enabling%20unidirectional%20supercurrent%20flow,30%20%2C%20%2048). This seminal result, with $I\_{c+}/I\_{c-}\approx 2.5$ (approximately 150% rectification), ignited widespread interest in superconducting diodes. Since then, JDEs have been realized across a variety of platforms, including non-centrosymmetric superconductors with strong SOC[[8]](https://arxiv.org/html/2502.11717v1#:~:text=heterostructures%20of%20%5BNb%2FV%2FTa%5Dn%20with%20out,47%2C%2031), Josephson junctions incorporating ferromagnets or Rashba semiconductors[[9]](https://arxiv.org/html/2502.11717v1#:~:text=superconducting%20systems%2C%20including%20chiral%20superconductors%C2%A0,47%2C%2031), and even all-superconducting devices under strain or asymmetric geometry[[10]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=%7BJ%7D_%7Bc%7D%5E%7B,and%20transition%20metal%20dichalcogenides%2044%2C16)[[11]](https://arxiv.org/html/2502.11717v1#:~:text=23%20%2C%20%2041%2C%2025,have%20been%20proven%20to%20be). In all cases, breaking **both** TRS and inversion symmetry has emerged as a general prerequisite for intrinsic diode behavior[[2]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=nonreciprocity,causes%20superconductor%20to%20normal%20metal)[[12]](https://arxiv.org/html/2502.11717v1#:~:text=superconductors%20with%20strong%20spin,TRS%20and%20IRS%20are%20generally). Notably, the superconducting diode effect provides a sensitive probe of exotic superconducting states—such as helical Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) states or chiral superconductivity—because it is directly tied to symmetry properties of the Cooper pairs[[13]](https://arxiv.org/html/2502.11717v1#:~:text=noncentrosymmetric%20superconductor%20,This%20review%20focuses)[[8]](https://arxiv.org/html/2502.11717v1#:~:text=heterostructures%20of%20%5BNb%2FV%2FTa%5Dn%20with%20out,47%2C%2031).

**Figure 1** below illustrates a typical device geometry implementing the Josephson diode effect. In this example, two superconducting electrodes form a junction through a noncentrosymmetric spacer layer that provides Rashba-type spin–orbit coupling, while an in-plane magnetic field $B$ (or an equivalently polarized ferromagnet) breaks TRS. The combination of broken inversion symmetry (due to the interfacial SOC in the spacer or device asymmetry) and the magnetic exchange/Zeeman field yields a directional dependence to the supercurrent. In such a setup, the positive critical current $I\_{c+}$ (for current flowing, say, left to right) can substantially exceed the negative critical current $I\_{c-}$[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands). Indeed, rectification efficiencies $(I\_{c+}-I\_{c-})/(I\_{c+}+I\_{c-})$ on the order of 30–40% have been both predicted and observed in optimized designs[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands)[[15]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=mechanisms%20of%20the%20SDE%20must,regarding%20the%20development%20of%20novel). Recent work demonstrated an **intrinsic zero-field SDE** (no external $B$) with $\sim40\%$ efficiency in a multilayer Nb/V/Ta superlattice containing interfacial Fe/Pt layers, where an internal exchange field and SOC conspire to break symmetries[[15]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=mechanisms%20of%20the%20SDE%20must,regarding%20the%20development%20of%20novel). These developments underscore the importance of understanding the theoretical underpinnings of the Josephson diode effect, which we address in this review.

**Figure 1: Device geometry (schematic) –** *A typical Josephson diode device consisting of two superconductors (S) coupled through a non-centrosymmetric spacer (N) with strong Rashba spin–orbit coupling. An in-plane magnetic field $B$ (or an embedded ferromagnet) provides time-reversal symmetry breaking. This configuration produces a finite-momentum pairing of Cooper pairs in the junction and an asymmetric critical current ($I\_{c+} \neq I\_{c-}$), realizing the superconducting diode effect. Arrows indicate the preferred direction of supercurrent flow.* ***(Placeholder for device geometry diagram)***

The remainder of this review is organized as follows. In **Section 2**, we discuss finite-momentum Cooper pairing mechanisms, providing the theoretical framework for how a finite pair momentum $q$ arises and leads to nonreciprocal supercurrents. We outline the mathematical formulations and link them to diode efficiency metrics, and we summarize experimental evidence for finite-$q$ pairing. **Section 3** focuses on Rashba-like spin–orbit coupling effects: how SOC-induced symmetry breaking contributes to the Josephson diode effect, the role of material interfaces and SOC strength, and quantitative estimates of SOC parameters in relation to diode behavior. In **Section 4**, we provide a comparative analysis of different theoretical models of the Josephson diode effect, including intrinsic (finite-$q$ and anomalous phase shift) and extrinsic (vortex/geomentry-based) mechanisms, summarized in a comparative table. Finally, **Section 5** offers a synthesis of the current understanding and discusses open questions and future directions, such as achieving higher diode efficiencies, exploring diode effects in new superconducting materials (e.g. topological or spin-triplet superconductors), and potential applications in superconducting circuits.

## 2. Finite-Momentum Cooper Pairing Mechanisms

### 2.1 Theoretical Foundations

In conventional BCS superconductors, Cooper pairs are formed by two electrons with opposite momenta ($\mathbf{k}$ and $-\mathbf{k}$) and opposite spins, resulting in a total center-of-mass momentum of zero. By contrast, **finite-momentum Cooper pairing (FMCP)** refers to pairing states where the Cooper pairs carry a nonzero total momentum $\mathbf{q}$. The possibility of such states was first pointed out in the 1960s by Fulde and Ferrell, and independently by Larkin and Ovchinnikov, in the context of ultrastrong magnetic fields[[16]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Shortly%20after%20the%20advent%20of,BCS%29%20theory%20of%20supercon). In the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase, a large Zeeman splitting of the Fermi surface can favor pairing between unequal Fermi momenta, yielding a spatially modulated order parameter $\Delta(\mathbf{r}) = \Delta\_{0}e^{i\mathbf{q}\cdot\mathbf{r}}$[[16]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Shortly%20after%20the%20advent%20of,BCS%29%20theory%20of%20supercon)[[17]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=this%20Fulde%E2%80%93Ferrell%E2%80%93Larkin%E2%80%93Ov%20chinnikov%20,of%20opposite%20spin%20states%20on). This exotic phase is separated from the standard $q=0$ BCS state by a first-order transition and requires extreme conditions (fields near the Pauli limit, low temperatures, and clean materials), making it challenging to observe directly[[18]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Zeeman,Extensive%20e%EE%98%82orts). Indeed, while indirect evidence of FFLO-like modulated superconductivity has been reported in certain materials via thermodynamic and NMR signatures[[18]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Zeeman,Extensive%20e%EE%98%82orts)[[19]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=dynamic%20and%20NMR%20measurements%20have,evidence%20of%20a%20distinctive%20superconducting), a clear demonstration of finite Cooper pair momentum remained elusive for decades.

Recently, a new route to finite-momentum pairing has been identified in **noncentrosymmetric superconductors** with strong spin–orbit coupling[[20]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Recently%2C%20a%20new%20type%20of,superconducting%20state%20has%20been%20predicted)[[21]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=magnetic%20%EE%98%83eld%2C%20a%20BCS%20superconductor,textured%20Fermi%20surfaces%20can%20smoothly). In such systems (typically two-dimensional electron gases or superconducting films lacking inversion symmetry), **Rashba SOC** splits the electronic band structure into two spin-polarized Fermi surfaces. The spin on each Fermi surface winds with momentum (forming a helical spin texture), as illustrated in **Figure 2a**. When an in-plane magnetic field $\mathbf{B}$ is applied, the two spin-split Fermi surfaces experience an additional momentum shift and Zeeman energy splitting[[20]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Recently%2C%20a%20new%20type%20of,superconducting%20state%20has%20been%20predicted). Crucially, because pairing can now occur **within** each single-spin Fermi surface (pairing of electrons with equal spin helicity), the superconducting state can continuously evolve into a finite-$q$ state under the field, without a first-order transition[[21]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=magnetic%20%EE%98%83eld%2C%20a%20BCS%20superconductor,textured%20Fermi%20surfaces%20can%20smoothly). This is often called a **helical superconducting state**, in which the order parameter acquires a phase modulation corresponding to a Cooper pair momentum $\mathbf{q}$ aligned with the direction of $\mathbf{B}$. In a simple picture, the field induces an imbalance in the occupation of states $+\mathbf{k}$ and $-\mathbf{k}$ on a given Fermi surface, so the Cooper pair formed from those states has momentum $\mathbf{q} \approx \mathbf{k}*{F+} - \mathbf{k}*$ proportional to the field strength.

**Figure 2: Band structure and finite-$q$ pairing –** *(a) Schematic band structure of a noncentrosymmetric superconductor with Rashba spin–orbit coupling. The spin–orbit interaction lifts spin degeneracy, producing two Fermi surfaces (solid circles) with opposite spin helicities (indicated by red and blue arrows). In the absence of a magnetic field, Cooper pairs form with zero center-of-mass momentum by pairing electrons from $+\mathbf{k}$ and $-\mathbf{k}$ on opposite Fermi surfaces (dashed lines). (b) Under an in-plane magnetic field $B$, the Fermi surfaces shift and spin states split in energy (illustrated by the offset circles). Cooper pairs can then form* within *a single Fermi surface (pairing indicated by curved arrow), acquiring a finite total momentum $\mathbf{q}$ (green arrow). This* *finite-momentum pairing* *leads to a spatially modulated order parameter $\Delta e^{i\mathbf{q}\cdot\mathbf{r}}$ and is the basis of the helical superconducting state responsible for the intrinsic Josephson diode effect.* ***(Placeholder for band structure diagram)***

The formation of a helical superconducting state with finite $\mathbf{q}$ has profound consequences for the Josephson current. Because the equilibrium superconducting condensate already carries a momentum $\mathbf{q}*{0}$ (set by the field and SOC), the supercurrent flowing through a junction will add or subtract from this condensate momentum depending on its direction. If a supercurrent flows* ***parallel*** *to the Cooper pair momentum $\mathbf{q}*}$, the total momentum of pairs is $(\mathbf{q*{0} + \delta\mathbf{q})$ (where $\delta\mathbf{q}$ relates to the superflow velocity); if current flows opposite, the pair momentum is $(\mathbf{q}*} - \delta\mathbf{q})$. The superconducting state will depair (lose zero-resistance superconductivity) when the total momentum magnitude $|\mathbf{q*{0}\pm \delta\mathbf{q}|$ exceeds a critical value set by the pair-breaking limit. Because $\mathbf{q}*\neq0$, the **depairing current** in one direction will reach this limit sooner than in the opposite direction[[22]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent)[[23]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=supercurrent,direct%20consequence%20of%20the%20Cooper). In other words, the **critical current is direction-dependent**: $I\_{c+}$ (current along $\mathbf{q}*{0}$) differs from $I*$) than the other[[22]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent)[[24]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=the). Their microscopic calculations predicted a sizeable diode effect as a direct consequence of the finite pair momentum breaking inversion and TRS in the ground state[[23]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=supercurrent,direct%20consequence%20of%20the%20Cooper).}$ (current opposite to $\mathbf{q}\_{0}$). This directional dependence is the essence of the supercurrent diode effect in a finite-$q$ superconductor[[22]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent)[[23]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=supercurrent,direct%20consequence%20of%20the%20Cooper). Yuan and Fu theoretically showed that in a 2D Rashba superconductor under in-plane field (a helical SC), the depairing critical current is larger in one direction (parallel to $\mathbf{q

The magnitude of the asymmetry can be quantified by a **diode efficiency** $\eta = (I\_{c+} - I\_{c-})/(I\_{c+} + I\_{c-})$. In the weak coupling and near-critical temperature limit, analytical Ginzburg–Landau (GL) theory can be used to estimate this asymmetry. Daido *et al.* derived that the amplitude of the critical current difference grows linearly with the equilibrium pair momentum $q\_{0}$ (and hence with applied field) near $T\_{c}$[[25]](https://arxiv.org/html/2502.11717v1#:~:text=pair,66%5D.%20Theories%20have%20also). Their GL analysis also predicted that the **polarity** of the diode effect (which direction is favored) can invert as field strength increases, providing a potential signature of an intrinsic SDE[[26]](https://arxiv.org/html/2502.11717v1#:~:text=dependence%20is%20manifested%20as%20SDEs,49%20%2C%20%2068%2C%2051). Microscopic calculations by Ilić and Bergeret further showed that a finite-$q$ diode effect persists even in the presence of strong disorder scattering[[27]](https://arxiv.org/html/2502.11717v1#:~:text=fields%2C%20which%20may%20serve%20as,49%20%2C%20%2068%2C%2051). Interestingly, disorder can even reverse the favored current direction as it alters the superconducting order parameter and effective pair momentum[[28]](https://arxiv.org/html/2502.11717v1#:~:text=match%20at%20L729%20,B%202022%2C%20106%2C%20205206). These theoretical results indicate that moderate nonmagnetic disorder does not destroy the diode effect—important for realistic devices—although it may reduce the overall efficiency.

The finite-momentum pairing mechanism typically yields a **phase-offset current–phase relation** in a Josephson junction. If one connects a finite-$q$ superconductor to a conventional superconductor, the Josephson energy is generally maximized at a phase difference $\phi = \phi\_{0}$ rather than $0$ or $\pi$. This reflects the built-in phase twist of the finite-$q$ condensate. More directly, in a short junction where the weak link itself supports a finite Cooper pair momentum, the CPR can acquire higher harmonics and an anomalous phase shift. Davydova *et al.* proposed a **universal mechanism** for JDE in short junctions wherein a finite momentum $\mathbf{q}$—achieved even without SOC by exploiting Meissner currents—causes a Doppler shift of Andreev bound state energies and a phase-independent asymmetric component of current[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands)[[29]](https://dspace.mit.edu/bitstream/handle/1721.1/146053/sciadv.abo0309.pdf?sequence=2&isAllowed=y#:~:text=,asymmetric%20current%20from%20the%20continuum). This produces a characteristic $\phi\_{0}$-shifted CPR of the form $I(\phi) = I\_{c}\sin(\phi - \phi\_{0})$. We will discuss such anomalous junction effects further in Section 4.

### 2.2 Experimental Evidence and Diode Efficiency

Several experiments have provided evidence consistent with finite-momentum pairing as the origin of observed supercurrent diode effects. The initial report by Ando *et al.* in 2020 involved a noncentrosymmetric multilayer (Nb/V/Ta) under an in-plane field[[5]](https://www.nature.com/articles/s41467-025-55880-4#:~:text=High,Jiang). The diode effect in that system (about 30% efficiency) was attributed to an inversion-symmetry-breaking superlattice potential combined with Zeeman splitting, an environment conducive to helical superconductivity[[7]](https://arxiv.org/html/2502.11717v1#:~:text=interface%2C%20enabling%20unidirectional%20supercurrent%20flow,30%20%2C%20%2048). Subsequently, **planar Josephson junctions** using materials with strong SOC have exhibited diode behavior: for instance, junctions with few-layer NbSe$\_2$ (superconductor with intrinsic Ising SOC) interfaced with a thin barrier (e.g. Nb$\_3$Br$\_8$ or an oxide) showed a field-free diode effect, presumably due to an internal exchange field at the interface[[30]](https://www.nature.com/articles/s41467-024-45298-9#:~:text=Intrinsic%20supercurrent%20non,reciprocal)[[31]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=superconductor%2Fferromagnet%20multilayers). In another example, Baumgartner *et al.* observed **supercurrent rectification** in symmetric lateral Josephson devices made from 2D materials, identifying magnetochiral anisotropy as the underlying mechanism[[32]](https://arxiv.org/pdf/2305.07923#:~:text=,17%2C%2039%20%282022). This is consistent with the finite-$q$ picture: the combination of structural asymmetry and an in-plane field (or magnetization) creates a preferred direction for supercurrent, matching theoretical predictions[[33]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=The%20inversion%20symmetry%20breaking%20required,60%2C28%20%2C%2062%2C30%20%2C%2064%2C32)[[34]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=Recent%20theoretical%20studies18%20%2C%2059%2C28,wave%20Cooper%20pairs6%20%2C%2058%2C28).

Experimental signatures of finite Cooper pair momentum in superconductors have also been reported via **critical field anisotropy**. In a helical SC, the in-plane upper critical field $H\_{c2}$ can differ depending on whether a bias current is applied parallel or antiparallel to $\mathbf{q}*{0}$*[*[22]*](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent)*. This manifests as a polarity-dependent critical field in transport measurements, another hallmark of finite-$q$ pairing*[*[22]*](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent)[*[24]*](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=the)*. On the junction level, the most direct evidence of asymmetric CPR is the measurement of different switching currents $I*$; this has now been documented in a range of systems including Al/InAs nanowire junctions with Rashba SOC (with an external $B$)[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self)[[36]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=comprehensive%2C%20self,81%2C46%20%2C%2083%2C48%20%2C%2085%2C50), and 2D trilayer graphene/NbSe$\_2$ devices where nonreciprocal supercurrents appear due to intrinsic polarization in the superconducting state[[37]](https://www.science.org/doi/10.1126/sciadv.ado1502#:~:text=Science%20www,induced%20electric%20polarization).}\neq I\_{c-

Quantitatively, reported **diode efficiencies** $\eta$ (at low temperature) have ranged from a few percent up to nearly 50% in some engineered structures[[38]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=FFLO%20pairing%20and%20highly%20efficient,This%20behavior). The theoretical upper bound for $\eta$ in a given system depends on how large a $q\_{0}$ can be induced before destroying superconductivity entirely. Davydova *et al.* estimated a maximum $\eta\approx40\%$ in a short Josephson junction with optimally induced $q\_{0}$[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands). This corresponds to a critical current asymmetry ratio $I\_{c+}/I\_{c-}\sim 2.3$[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands). Notably, Narita *et al.* achieved $\eta\approx40\%$ in a zero-field S/F superlattice device by tuning the magnetization direction[[15]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=mechanisms%20of%20the%20SDE%20must,regarding%20the%20development%20of%20novel)[[39]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=study%20demonstrates%20an%20intrinsic%20zero,Moreover%2C%20the), indicating that such high efficiencies are indeed reachable. On the other hand, simpler Rashba semiconducting weak links under moderate fields often show more modest efficiencies (a few to 10%) unless carefully optimized[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self). Self-consistent theoretical modeling has shown that accounting for suppression of the order parameter by the induced momentum (the superconducting depairing effect) is crucial for accurately predicting diode efficiencies[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self)[[40]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=systematically%20investigate%20the%20superconducting%20order,the%20parameters%20in%20this%20system). By optimizing parameters such as the SOC strength, $g$-factor (Zeeman coupling), and electron density, efficiencies above 40% have been predicted even in one-dimensional Rashba nanowire devices[[36]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=comprehensive%2C%20self,81%2C46%20%2C%2083%2C48%20%2C%2085%2C50)[[38]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=FFLO%20pairing%20and%20highly%20efficient,This%20behavior). Moreover, certain unconventional scenarios could host a **“perfect” diode effect** ($\eta\to100\%$). For example, theoretical work on *altermagnets* (magnetic materials with zero net magnetization but momentum-dependent spin splitting) suggests that an *FFLO-like pairing state* can arise without an external field, potentially allowing one direction of supercurrent to be completely blocked while the opposite remains dissipationless[[41]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=that%20is%20widely%20believed%20to,dependent%20oscillations%20in%20the)[[42]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=supercurrent%2C%20%28iii%29%20large,8). While a perfect superconducting diode is mostly theoretical at this stage, these studies highlight the rich possibilities when combining finite-momentum superconductivity with complex band structures.

**Figure 3: Current–phase relation asymmetry –** *Illustration of an anomalous current–phase relation (CPR) in a Josephson junction exhibiting the diode effect. The solid curve shows a CPR $I(\phi)$ that is skewed and shifted by an angle $\varphi\_{0}$ (dashed line indicates the phase shift from $\phi=0$). Here $I\_{c+}$ and $I\_{c-}$ denote the magnitudes of critical current in the positive and negative directions, respectively. The asymmetry ($I\_{c+} > I\_{c-}$ in this example) arises from a second-harmonic component and phase shift in the CPR, induced by finite-momentum pairing and spin–orbit coupling. The non-zero current at $\phi=0$ (anomalous Josephson current) is another signature of broken time-reversal and inversion symmetries. This CPR shape corresponds to a $\varphi\_{0}$-junction, and the area under the curve (shaded) differs for forward vs reverse bias, indicating net diode-like behavior.* ***(Placeholder for current–phase relation diagram)***

## 3. Rashba Spin–Orbit Coupling Contributions

### 3.1 SOC-Induced Symmetry Breaking Mechanisms

**Rashba spin–orbit coupling (SOC)** plays a central role in many realizations of the Josephson diode effect by providing a built-in mechanism to break inversion symmetry. Rashba SOC typically arises in structures lacking inversion symmetry (for example, at surfaces or interfaces, or in materials with a polar crystal structure). It can be described by a Hamiltonian term of the form $H\_{\mathrm{SOC}} = \alpha (\mathbf{p} \times \boldsymbol{\sigma})\cdot \hat{\mathbf{z}}$, where $\alpha$ is the Rashba coefficient (proportional to the interfacial electric field or atomic spin–orbit strength), $\mathbf{p}$ is the electron momentum, $\boldsymbol{\sigma}$ are the Pauli spin matrices, and $\hat{\mathbf{z}}$ is the axis of inversion asymmetry. In a two-dimensional electron gas (2DEG) with Rashba SOC, this leads to an electronic dispersion:

$$E\_{\pm}(k) = \frac{\hbar^2 k^2}{2m} \pm \alpha k,$$

for an isotropic effective mass $m$ (valid at low $k$). The $\pm$ signs correspond to two spin-split branches (often called Rashba bands) whose spin orientations are opposite for a given momentum direction. The result is a momentum-dependent spin polarization, i.e. a **helical spin texture**, on the Fermi surfaces (as depicted in Figure 2a). Importantly, Rashba SOC **breaks inversion symmetry** because the relation $E\_{\uparrow}(k) = E\_{\downarrow}(-k)$ no longer holds globally—spin and momentum are locked in one orientation. Time-reversal symmetry, however, remains intact in the pure Rashba system *unless* an additional magnetic field or exchange interaction is introduced. This means that by itself, SOC induces a form of **magnetochiral anisotropy (MCA)**: charge transport can become nonreciprocal in the presence of a magnetic field, even though neither inversion nor TRS alone would be sufficient[[2]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=nonreciprocity,causes%20superconductor%20to%20normal%20metal)[[43]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=the%20regime%20where%20the%20critical,and%20transition%20metal%20dichalcogenides%2044%2C16). In the normal (non-superconducting) state, this manifests as a small difference in resistance for opposite current directions when a magnetic field is applied (the magnetochiral effect). When superconductivity is present, the effect of SOC + magnetic field is greatly amplified because the superconducting coherence makes the system more sensitive to symmetry breaking[[44]](https://arxiv.org/html/2502.11717v1#:~:text=Nonreciprocal%20charge%20transport%20also%20occurs,that%20in%20the%20normal%20state). In fact, the superconducting diode effect can be viewed as an extreme limit of magnetochiral anisotropy: zero resistance for current in one direction and finite resistance (or a normal state transition) in the opposite direction[[45]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=non,Recent%20experiments%20in%20engineered%20superlattice)[[10]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=%7BJ%7D_%7Bc%7D%5E%7B,and%20transition%20metal%20dichalcogenides%2044%2C16).

From a symmetry perspective, Rashba SOC lowers the point-group symmetry of the system (for instance, from centrosymmetric to polar point group). This allows the superconducting order parameter to couple to magnetic or current-carrying states in ways forbidden in centrosymmetric superconductors. For example, in a Rashba superconductor under a magnetic field, an **antisymmetric supercurrent** term is permitted in Ginzburg–Landau free energy expansions[[46]](https://arxiv.org/html/2502.11717v1#:~:text=space%20%2C%20where%20is%20the,Moreover%2C%20Ili%C4%87)[[25]](https://arxiv.org/html/2502.11717v1#:~:text=pair,66%5D.%20Theories%20have%20also). This leads to a superconducting state carrying a spontaneous supercurrent (finite $\mathbf{q}$) even in equilibrium, as described in Section 2. In contrast, in a centrosymmetric superconductor, any supercurrent in equilibrium would violate Onsager reciprocity unless TRS is also broken. Thus, Rashba SOC provides one half of the recipe for nonreciprocal supercurrents: it **breaks inversion symmetry**, while a magnetic field or exchange bias **breaks TRS**, and together they produce a diode effect[[33]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=The%20inversion%20symmetry%20breaking%20required,60%2C28%20%2C%2062%2C30%20%2C%2064%2C32)[[1]](https://arxiv.org/html/2502.11717v1#:~:text=the%20Onsager%20reciprocity%20relations%2C%20this,that%20in%20the%20normal%20state). Notably, if either symmetry is restored (e.g. no SOC or no magnetic ordering), the diode effect vanishes and $I\_{c+}=I\_{c-}$ by symmetry.

### 3.2 Interface and Material-Dependent Effects

The strength and nature of spin–orbit coupling can vary widely depending on the materials and interfaces involved, which in turn influences the Josephson diode effect. **Rashba-like SOC** is often engineered using heavy metal layers or substrates (such as Pt, Au, or topological insulators) in proximity to superconductors or semiconductors. The magnitude of the Rashba coefficient $\alpha$ can range from $10^{-11}$ eV·m in typical semiconductor heterostructures to above $10^{-10}$ eV·m in interfaces with strong heavy-metal involvement. A useful characteristic length scale is the **spin-orbit length** $l\_{\mathrm{so}} = \hbar^2/(m\alpha)$. If $l\_{\mathrm{so}}$ is comparable to or shorter than the superconducting coherence length $\xi$, spin–orbit effects strongly influence the pairing and current-phase relations. For instance, in an InAs 2DEG with $\alpha \approx 1$–$5\times10^{-11}$ eV·m and $\xi\sim 200$ nm, $l\_{\mathrm{so}}\sim 100$ nm, meaning SOC is a significant perturbation to Cooper pairs[[34]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=Recent%20theoretical%20studies18%20%2C%2059%2C28,wave%20Cooper%20pairs6%20%2C%2058%2C28)[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self). In contrast, in a low-SOC material $l\_{\mathrm{so}}$ would be much larger than $\xi$, and inversion symmetry breaking would have a weaker impact on superconducting properties.

At material interfaces, **structural asymmetry** (such as a substrate vs vacuum interface) can induce Rashba splitting even in materials with negligible bulk SOC. The **interface quality and inversion asymmetry gradient** (e.g. atomic layer arrangement or gating electric fields) thus directly set $\alpha$ and can be tuned in experiments. This has been demonstrated by gate-controlled SOC strength in semiconductor nanowires, which modulates the observed diode efficiency[[47]](https://www.nature.com/articles/s42005-025-02044-x#:~:text=interplay%20of%20Rashba%20spin,even%20without%20longitudinal%20magnetic%20fields)[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self). Furthermore, different types of SOC can coexist: for example, Dresselhaus SOC (arising from bulk crystal inversion asymmetry in certain III–V semiconductors) or Ising SOC (as in few-layer NbSe$\_2$, where spin splitting is out-of-plane) may contribute alongside Rashba. These can produce complex spin-splitting landscapes. A recent study in proximitized nanowires showed that including **higher-order (cubic) Rashba terms** (from bulk inversion asymmetry) qualitatively changes the diode effect behavior[[38]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=FFLO%20pairing%20and%20highly%20efficient,This%20behavior)[[48]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=large%20diode%20efficiency%20%28%20%E2%89%B345,Our%20study%20establishes%20the). Specifically, while a linear Rashba SOC by itself requires an external longitudinal field to generate a $q\neq0$, the presence of cubic SOC terms can induce a finite pair momentum even without a longitudinal field, resulting in a nonzero diode efficiency $\eta \sim15\%$ at $B=0$ in theory[[48]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=large%20diode%20efficiency%20%28%20%E2%89%B345,Our%20study%20establishes%20the). This surprising result implies that certain crystalline directions or band structures might allow an **intrinsic SDE without net field**, if higher-order spin–orbit interactions effectively break time-reversal symmetry in the superconducting state (for example, via a parity-mixed order parameter or odd-parity pairing component). Another material example is the case of **altermagnets** (collinear magnets with zero net magnetization but momentum-dependent spin splitting). In a recent theoretical work, a superconductor proximitized to an altermagnet was predicted to exhibit finite-momentum pairing despite *zero average magnetization*, with a highly anisotropic $\mathbf{q}(\theta)$ that depends on the propagation direction in the crystal[[41]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=that%20is%20widely%20believed%20to,dependent%20oscillations%20in%20the)[[49]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=Remarkably%2C%20we%20find%20that%20the,junctions%20oriented%20along%20different%20directions). This leads to unusual orientation-dependent Josephson effects, including 0–$\pi$ transitions and diode-like behavior that can be toggled by device geometry[[50]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=several%20unique%20features%3A%20,8)[[51]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=Fig.%201%3A%20Contrast%20of%20finite,the%20ferromagnetic%20and%20altermagnetic%20metals). Such findings underscore the **material-dependence** of SOC effects: the symmetry of the underlying electronic structure (whether simple Rashba, cubic Dresselhaus, or more exotic spin-splitting) will dictate how a magnetic perturbation translates into a Cooper pair momentum and hence a diode effect.

Experimentally, interfaces with heavy elements (e.g. a thin Pt insertion layer in Nb/V/Ta superlattices[[15]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=mechanisms%20of%20the%20SDE%20must,regarding%20the%20development%20of%20novel), or a Bi$\_2$Te$\_3$ topological insulator surface on which superconductivity is induced[[52]](https://link.aps.org/doi/10.1103/PhysRevB.109.L081405#:~:text=states%20link,of%20the%20Josephson%20diode%20effect)) have yielded some of the most robust diode effects, confirming that strong SOC promotes nonreciprocity. In the Nb/V/Ta artificial superlattice with Fe/Pt layers mentioned earlier, first-principles calculations indicated that an **asymmetric distribution of induced magnetic moments** in the superconducting layers (an effect of the Fe/Pt interface) is correlated with the diode effect[[39]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=study%20demonstrates%20an%20intrinsic%20zero,Moreover%2C%20the)[[53]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=superlattices,regarding%20the%20development%20of%20novel). Essentially, the interface SOC and magnetism create a **magnetic toroidal moment** within the superconductor, breaking both inversion and TRS in a subtle but effective manner[[39]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=study%20demonstrates%20an%20intrinsic%20zero,Moreover%2C%20the). This points to an interesting materials-design principle: combining ferromagnets and heavy metals in a noncentrosymmetric stack can generate *internal* fields and spin–orbit fields that act on Cooper pairs, potentially eliminating the need for an external magnetic field to achieve the diode effect.

### 3.3 Quantitative Analysis of SOC Strength

A quantitative understanding of how SOC strength impacts the Josephson diode effect can be gleaned from simplified theoretical models. In a Rashba SOC nanowire model, for instance, the induced Cooper pair momentum in a helical superconducting state is roughly proportional to the ratio of the Zeeman energy to the Fermi velocity in each spin band. If $\mu$ is the chemical potential and $\Delta$ the superconducting gap, one finds that the pair momentum $q\_{0}$ satisfies (to first order in field $B$) $q\_{0} \approx \frac{g\mu\_{B}B}{\hbar v\_{F}} \frac{\alpha k\_{F}}{\sqrt{\mu^2 + \Delta^2}}$, where $g\mu\_{B}B$ is the Zeeman energy and $\alpha k\_{F}$ is the spin–orbit splitting at the Fermi level[[34]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=Recent%20theoretical%20studies18%20%2C%2059%2C28,wave%20Cooper%20pairs6%20%2C%2058%2C28)[[54]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=resulting%20in%20Kramers%20non,Cooper%20pairs6%20%2C%2058%2C28). This expression indicates that a larger SOC-induced splitting (larger $\alpha k\_{F}$) directly leads to a larger $q\_{0}$ for a given magnetic field, thus enhancing the diode effect. It also suggests that materials with low Fermi velocity $v\_{F}$ or low carrier density (so that $k\_{F}$ is small) can achieve larger $q\_{0}$ at lower fields, which is advantageous for realizing SDE without destroying superconductivity. These considerations align with experimental trends: diode efficiencies tend to be higher in devices with lower carrier densities and stronger SOC (e.g. in few-layer or low-$k\_F$ systems)[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self)[[40]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=systematically%20investigate%20the%20superconducting%20order,the%20parameters%20in%20this%20system).

Another useful parameter is the ratio of the spin-orbit energy $E\_{\mathrm{so}} = \frac{m\alpha^2}{2\hbar^2}$ to the superconducting gap $\Delta$. If $E\_{\mathrm{so}} \gg \Delta$, the spin splitting of the normal state bands is large compared to the pairing scale, meaning the superconducting state will inherently reflect the spin–momentum locking (often yielding a strong tendency toward finite-$q$ pairing under any perturbation). In contrast, if $E\_{\mathrm{so}} \ll \Delta$, the superconductivity "pairs across" the spin-split bands and might behave more like a conventional $s$-wave, less sensitive to SOC unless a large field is applied. Thus, **strong SOC regimes (large $\alpha$)** are generally favorable for observing intrinsic JDE. For instance, in one study of a proximitized InSb nanoflag Josephson junction (with large $\alpha$ and $g$-factor), a significant SDE at zero field was reported, attributed to the intrinsically spin–orbit-coupled band structure and a built-in inversion asymmetry in the device design[[55]](https://www.bohrium.com/paper-details/josephson-transistor-from-the-superconducting-diode-effect-in-domain-wall-and-skyrmion-magnetic-racetracks/936280475549302786-98281#:~:text=Josephson%20transistor%20from%20the%20superconducting,org).

One must also consider higher-order spin–orbit effects quantitatively. As mentioned, inclusion of **cubic Rashba** or **Dresselhaus** terms leads to additional terms in the pair-breaking current asymmetry. Hasan *et al.* (2024) analyzed a 2D Rashba-Zeeman model with both linear and cubic SOC and found that the diode efficiency depends non-monotonically on the relative strength of these terms[[56][57]](https://arxiv.org/html/2502.11717v1#:~:text=,B%202022%2C%20106%2C%20205206). In certain regimes, cubic SOC can even dominate the sign of $\eta$. These analyses highlight that extracting a simple figure of merit for SOC strength in complex materials can be challenging; one often needs self-consistent numerical calculations to predict $\eta$ accurately for a given band structure[[40]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=systematically%20investigate%20the%20superconducting%20order,the%20parameters%20in%20this%20system)[[36]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=comprehensive%2C%20self,81%2C46%20%2C%2083%2C48%20%2C%2085%2C50). Nonetheless, as a rule of thumb, **increasing SOC strength $\alpha$** (or using materials with heavier elements and stronger intrinsic SOC) tends to **increase the diode efficiency**, up to the point where other pair-breaking effects (like orbital depairing by field or interface roughness) become limiting factors[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self)[[38]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=FFLO%20pairing%20and%20highly%20efficient,This%20behavior).

In summary, Rashba-like SOC is a critical ingredient in most intrinsic Josephson diode effect scenarios, as it provides the necessary inversion-symmetry breaking and spin-split Fermi surfaces for finite-$q$ pairing. The effectiveness of SOC in inducing diode behavior is strongly material- and interface-dependent, and maximizing the effect calls for large SOC energy scales and careful balancing of magnetic perturbations. In the next section, we compare various theoretical approaches to the Josephson diode effect, including those centered on SOC-induced finite momentum pairing and other mechanisms that have been proposed.

## 4. Comparative Analysis of Theoretical Models

A number of theoretical frameworks have been developed to explain the supercurrent diode effect, each emphasizing different physical mechanisms. Here, we compare the primary approaches, highlighting their key features, required symmetries, and predictions. **Table 1** provides a summary of these approaches: (i) finite-momentum Cooper pairing in the superconducting state, (ii) anomalous Josephson coupling leading to an unconventional current–phase relation, and (iii) extrinsic mechanisms related to vortex dynamics or device geometry. This comparative view underscores that while the *symmetry breaking condition* (inversion + TRS broken) is common to all, the microscopic origin of the asymmetry can differ.

# table(

| l | l | l | | **Theoretical Approach** | **Mechanism & Symmetry Breaking** | **Key Characteristics & Predictions** | | **Finite-momentum Cooper pairing** (helical/FFLO-like superconductivity) | Cooper pairs carry a finite center-of-mass momentum $\mathbf{q}$ in equilibrium due to broken inversion (e.g. Rashba SOC) and an exchange or Zeeman field (broken TRS). The order parameter is modulated as $\Delta(\mathbf{r}) = \Delta\_0 e^{i\mathbf{q}\cdot\mathbf{r}}$. | Intrinsic mechanism requiring both inversion and TRS broken in the superconducting state[[34]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=Recent%20theoretical%20studies18%20%2C%2059%2C28,wave%20Cooper%20pairs6%20%2C%2058%2C28)[[1]](https://arxiv.org/html/2502.11717v1#:~:text=the%20Onsager%20reciprocity%20relations%2C%20this,that%20in%20the%20normal%20state). Yields asymmetric depairing currents $I\_{c+}\neq I\_{c-}$[[22]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent). Diode efficiency can be substantial (up to $\sim40$–50% in ideal cases)[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands). Signature predictions: polarity-dependent $H\_{c2}$ and a phase shift in Josephson current. Exemplary works: Yuan & Fu (2022) helical SC model[[22]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=In%20this%20work%2C%20we%20show,superconductors%20exhibit%20an%20intrinsic%20supercurrent); Davydova *et al.* (2022) short JJ with finite-$q$ pairing[[14]](https://arxiv.org/abs/2201.00831#:~:text=,coupling%20and%20thus%20greatly%20expands). | | **Anomalous CPR ($\varphi\_{0}$-junction)** | Josephson junction with intrinsic SOC and a magnetic moment (exchange field) exhibits an unconventional current–phase relation: $I(\phi) = I\_{1}\sin\phi + I\_{2}\sin(2\phi) + \cdots$, including a phase shift $\varphi\_{0}\neq0$ (an **anomalous Josephson effect**)[[58]](https://arxiv.org/html/2502.11717v1#:~:text=across%20two%20superconductors%20is%20known,asymmetric%20properties%20related%20to%20the). This arises from a phase-biased energy minimum shifted by SOC–Zeeman coupling. | Reflects simultaneous inversion and TRS breaking at the junction level (e.g. in S/F/S junctions with Rashba interfaces)[[58]](https://arxiv.org/html/2502.11717v1#:~:text=across%20two%20superconductors%20is%20known,asymmetric%20properties%20related%20to%20the). Manifests as a finite Josephson current at zero phase ($I(\phi=0)\neq0$) and a skewed CPR[[59]](https://arxiv.org/html/2502.11717v1#:~:text=considering%20the%20presence%20of%20Rashba,Davydova%20put%20forth%20a%20simple). Leads to different positive/negative critical currents even if the bulk superconductors have $q=0$. Often accompanies finite-$q$ states or appears as $\phi\_{0}$-shift in helical SC[[60]](https://arxiv.org/html/2502.11717v1#:~:text=Notably%2C%20this%20scalar%20product%20reveals,momentum%20Cooper%20pairs%C2%A0%5B64). Predicted to cause diode effect in short junctions (phase shift $\varphi\_{0}$ enhances $I\_{c+}-I\_{c-}$)[[60]](https://arxiv.org/html/2502.11717v1#:~:text=Notably%2C%20this%20scalar%20product%20reveals,momentum%20Cooper%20pairs%C2%A0%5B64). Example: Buzdin and others predicted $\varphi\_{0}$-junctions with Rashba + magnetic exchange (2008); experimentally observed in nanowire JJs (2016). Davydova’s mechanism (2022) can be viewed in this class, where Doppler-shifted Andreev states yield an effective $\varphi\_{0}$[[60]](https://arxiv.org/html/2502.11717v1#:~:text=Notably%2C%20this%20scalar%20product%20reveals,momentum%20Cooper%20pairs%C2%A0%5B64). | | **Vortex/Extrinsic mechanisms** (asymmetric kinetics) | Spatially asymmetric device features (geometry, defects) cause unequal flux or vortex entry for opposite current directions. For example, a thickness gradient or array of nanoholes breaks inversion symmetry in how vortices nucleate at edges[[61]](https://arxiv.org/html/2502.11717v1#:~:text=In%20the%20context%2C%20the%20initial,creates%20an%20asymmetric%20vortex%20pinning). The self-induced magnetic field of the transport current (or a small residual field) breaks TRS when combined with the asymmetry. | Considered an **extrinsic** diode effect: not arising from an intrinsic superconducting order parameter symmetry, but from current-driven vortex dynamics[[61]](https://arxiv.org/html/2502.11717v1#:~:text=In%20the%20context%2C%20the%20initial,creates%20an%20asymmetric%20vortex%20pinning)[[62]](https://arxiv.org/html/2502.11717v1#:~:text=shown%20in%20Figure%C2%A01%20c%C2%A0,for%20the%20observed%20high%20rectification). Nonreciprocity appears as one polarity reaching the critical state (vortex penetration) at a lower current than the other. Key characteristics: often only observed in the mixed state (type-II superconductors), disappears if vortices are absent. Can yield large rectification but is sensitive to pinning and inhomogeneity[[62]](https://arxiv.org/html/2502.11717v1#:~:text=shown%20in%20Figure%C2%A01%20c%C2%A0,for%20the%20observed%20high%20rectification)[[63]](https://arxiv.org/html/2502.11717v1#:~:text=The%20vortex,54). Example realizations: stepped NbSe$\_2$ films (one edge thinner) showing diode-like $I\_c$ due to asymmetric vortex surface barrier[[64]](https://arxiv.org/html/2502.11717v1#:~:text=significantly%20affected%20by%20asymmetrical%20vortex,to%20the%20formation%20of%20asymmetric); MoGe microbridge with a designed asymmetric pinning array exhibiting high rectification efficiency[[62]](https://arxiv.org/html/2502.11717v1#:~:text=shown%20in%20Figure%C2%A01%20c%C2%A0,for%20the%20observed%20high%20rectification). While extrinsic SDEs can be tuned via device design and are useful for applications, they do not directly probe novel superconducting pairing states[[65]](https://arxiv.org/html/2502.11717v1#:~:text=response%20at%20zero%20field%C2%A0,as%20depicted%20in%20Figure%C2%A0%2063e)[[63]](https://arxiv.org/html/2502.11717v1#:~:text=The%20vortex,54). | )

**Table 1:** *Comparison of major theoretical approaches to the Josephson diode effect.* Each model requires broken inversion symmetry (IRS) and time-reversal symmetry (TRS) but differs in the microscopic origin of nonreciprocity. Finite-momentum pairing is an intrinsic bulk mechanism tied to the superconducting order parameter acquiring momentum; anomalous CPR mechanisms focus on junction-level phase shifts (often coexisting with finite $q$ in practice); vortex and extrinsic mechanisms rely on asymmetric current-induced dissipation (extrinsic to the condensate). The table highlights representative features and references for each approach.

As seen above, the **finite-momentum pairing** and **anomalous CPR** viewpoints are closely related and often both present in a given device: a junction hosting a helical superconductor will naturally exhibit a $\varphi\_{0}$-shifted CPR, combining elements of the first two approaches. Both are considered “intrinsic” since they stem from the superconducting order parameter’s symmetry. In contrast, **extrinsic vortex mechanisms** do not require unconventional pairing—indeed, they can occur in a conventional superconductor with no exotic order—so long as there is a structural asymmetry. One practical consequence is that extrinsic effects can mask intrinsic diode behavior; experiments must carefully distinguish whether a measured $I\_{c+}\neq I\_{c-}$ is due to vortex pinning asymmetry or due to an underlying $\varphi\_{0}$ junction or finite-$q$ state[[65]](https://arxiv.org/html/2502.11717v1#:~:text=response%20at%20zero%20field%C2%A0,as%20depicted%20in%20Figure%C2%A0%2063e). Techniques such as measuring the field-polarity dependence, temperature scaling, or examining the shape of $I(\phi)$ can help identify the mechanism. For instance, an intrinsic diode effect might reverse sign when an external field is reversed (since $\mathbf{q}$ would flip direction), whereas an extrinsic geometric diode effect tied to sample asymmetry might remain the same under field reversal (if the physical asymmetry is unchanged)[[66]](https://arxiv.org/html/2502.11717v1#:~:text=of%20asymmetric%20critical%20current%20follows,67%2C%2050%20%2C%20%2069).

In terms of **efficiency**, intrinsic mechanisms are generally expected to scale with superconducting parameters (gap, SOC strength, etc.) and can potentially achieve higher efficiencies in the clean limit[[36]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=comprehensive%2C%20self,81%2C46%20%2C%2083%2C48%20%2C%2085%2C50). Extrinsic mechanisms might allow high efficiency in specially engineered structures but could be more vulnerable to material imperfections. From a device standpoint, both approaches are being explored: intrinsic diodes for fundamental studies and possibly more reproducible behavior linked to quantum phenomena, and extrinsic diodes for easier engineering of rectification in superconducting circuits.

## 5. Conclusions and Outlook

The Josephson diode effect represents a fundamentally new way to achieve directionality in dissipationless electronics, enabled by the interplay of symmetry breaking and superconductivity. In this review, we have examined the two primary theoretical pillars of the JDE: finite-momentum Cooper pairing mechanisms and Rashba-like spin–orbit coupling contributions. The finite-$q$ pairing picture provides a natural explanation for intrinsic diode effects as a consequence of a helical superconducting ground state that breaks inversion and time-reversal symmetry simultaneously[[23]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=supercurrent,direct%20consequence%20of%20the%20Cooper)[[34]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=Recent%20theoretical%20studies18%20%2C%2059%2C28,wave%20Cooper%20pairs6%20%2C%2058%2C28). We traced the development of this idea from the FFLO state in high-field superconductors to more recent predictions of smoothly induced finite momentum in noncentrosymmetric systems[[20]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=Recently%2C%20a%20new%20type%20of,superconducting%20state%20has%20been%20predicted)[[21]](https://www.researchgate.net/publication/359725010_Supercurrent_diode_effect_and_finite-momentum_superconductors#:~:text=magnetic%20%EE%98%83eld%2C%20a%20BCS%20superconductor,textured%20Fermi%20surfaces%20can%20smoothly). The Rashba SOC framework, on the other hand, highlights how materials and interfaces supplying strong spin–orbit interactions can serve as the symmetry-breaking engine that, when combined with a magnetic exchange or field, drives the system into a finite-$q$ or $\varphi\_{0}$-shifted state[[58]](https://arxiv.org/html/2502.11717v1#:~:text=across%20two%20superconductors%20is%20known,asymmetric%20properties%20related%20to%20the)[[60]](https://arxiv.org/html/2502.11717v1#:~:text=Notably%2C%20this%20scalar%20product%20reveals,momentum%20Cooper%20pairs%C2%A0%5B64). Both perspectives are not mutually exclusive but rather complementary: Rashba SOC creates the conditions for finite-momentum pairing, and finite-momentum pairing (or equivalently a $\varphi\_{0}$ CPR) is the manifestation of those conditions in the superconducting order parameter.

**Current understanding:** The theoretical models discussed (and summarized in Table 1) have successfully explained many experimental observations of the JDE across different platforms. There is a consensus that breaking both key symmetries (TRS and inversion) is essential in all intrinsic cases[[1]](https://arxiv.org/html/2502.11717v1#:~:text=the%20Onsager%20reciprocity%20relations%2C%20this,that%20in%20the%20normal%20state)[[12]](https://arxiv.org/html/2502.11717v1#:~:text=superconductors%20with%20strong%20spin,TRS%20and%20IRS%20are%20generally). Finite-momentum pairing provides a unifying framework that connects the JDE to other phenomena like the anomalous Josephson effect (where a $\varphi\_{0}$ phase shift occurs) and magnetochiral anisotropy in transport[[45]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=non,Recent%20experiments%20in%20engineered%20superlattice)[[10]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=%7BJ%7D_%7Bc%7D%5E%7B,and%20transition%20metal%20dichalcogenides%2044%2C16). The critical current asymmetry has been quantitatively linked to microscopic parameters: increasing SOC strength, magnetic exchange, or utilizing materials with large spin-splitting generally enhances the effect[[38]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=FFLO%20pairing%20and%20highly%20efficient,This%20behavior)[[48]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=large%20diode%20efficiency%20%28%20%E2%89%B345,Our%20study%20establishes%20the), whereas disorder or suboptimal Fermi level alignment may suppress it[[28]](https://arxiv.org/html/2502.11717v1#:~:text=match%20at%20L729%20,B%202022%2C%20106%2C%20205206)[[56]](https://arxiv.org/html/2502.11717v1#:~:text=,B%202022%2C%20106%2C%20205206). Experimentally, the ability to achieve substantial diode efficiencies (20–40% or more) without an external magnetic field—either via internal magnetism or clever device design—marks a significant advancement[[15]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=mechanisms%20of%20the%20SDE%20must,regarding%20the%20development%20of%20novel)[[39]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=study%20demonstrates%20an%20intrinsic%20zero,Moreover%2C%20the). This paves the way for integrating superconducting diodes into superconducting circuits (for example, as rectifiers or non-dissipative current selectors in logic architectures).

**Open questions:** Despite the progress, numerous questions remain open for future investigation. One important question is **how close to 100% rectification** can an intrinsic superconducting diode effect get? Theoretical proposals like those involving altermagnets or multi-component order parameters suggest that perfect or near-perfect diode action might be achievable in principle[[41]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=that%20is%20widely%20believed%20to,dependent%20oscillations%20in%20the)[[42]](https://www.nature.com/articles/s41467-024-45951-3?error=cookies_not_supported&code=e013f87f-3352-4bb9-be66-63e32dcb2e01#:~:text=supercurrent%2C%20%28iii%29%20large,8). Realizing such conditions experimentally will require identifying materials where a single direction of supercurrent is completely suppressed. Another avenue of inquiry is the **temperature dependence and robustness** of the diode effect: many experiments show $\eta$ decreasing as $T$ approaches $T\_c$, consistent with GL theory[[26]](https://arxiv.org/html/2502.11717v1#:~:text=dependence%20is%20manifested%20as%20SDEs,49%20%2C%20%2068%2C%2051), but detailed studies of the diode effect in the vicinity of $T\_c$ (including fluctuations above $T\_c$[[1]](https://arxiv.org/html/2502.11717v1#:~:text=the%20Onsager%20reciprocity%20relations%2C%20this,that%20in%20the%20normal%20state)[[44]](https://arxiv.org/html/2502.11717v1#:~:text=Nonreciprocal%20charge%20transport%20also%20occurs,that%20in%20the%20normal%20state)) could shed light on the fundamental limits and the role of superconducting fluctuations.

The **dynamics and AC response** of Josephson diodes is another largely unexplored area. For instance, how does a Josephson diode behave under high-frequency or microwave excitation? The interplay of AC Josephson currents and nonreciprocity might enable novel functionalities like superconducting diode mixers or circulators. Theories that include time-dependent Ginzburg–Landau equations or Josephson junction dynamics will be needed to explore these regimes. Furthermore, the question of **switchability and controllability** of the diode effect is pertinent for applications: can one tune or reverse the diode polarity *in situ*? Some experiments have demonstrated magnetization-direction control of the diode polarity[[39]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=study%20demonstrates%20an%20intrinsic%20zero,Moreover%2C%20the)[[53]](https://pubmed.ncbi.nlm.nih.gov/37410358/#:~:text=superlattices,regarding%20the%20development%20of%20novel), and electrostatic gating has been used to modulate or even invert the diode effect in certain junctions[[35]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=A%20Rashba%20nanowire%20device%20proximitized,superconducting%20order%20parameter%20using%20self)[[67]](https://www.nature.com/articles/s42005-025-02044-x?error=cookies_not_supported&code=ec4438fc-2e22-46b4-9c71-4caeca4b91df#:~:text=helical%20superconductivity25%20%2C%2070%2C35%20%2C,potentially%20uncover%20pathways%20for%20improvement). Developing a theoretical understanding of gating effects on $\varphi\_{0}$ and $q$ (perhaps through electric-field modulation of SOC or carrier density) will be useful for designing voltage-controlled Josephson diodes.

Another exciting direction is the exploration of **topological superconductors and Josephson diodes**. If a junction hosts Majorana bound states or other topological features, could it exhibit an intrinsic diode effect or a related nonreciprocal response[[68]](https://link.aps.org/doi/10.1103/PhysRevB.109.L081405#:~:text=Enhancing%20the%20Josephson%20diode%20effect,of%20the%20Josephson%20diode%20effect)[[52]](https://link.aps.org/doi/10.1103/PhysRevB.109.L081405#:~:text=states%20link,of%20the%20Josephson%20diode%20effect)? Initial works have suggested that Majorana zero modes could enhance the diode effect in some cases[[68]](https://link.aps.org/doi/10.1103/PhysRevB.109.L081405#:~:text=Enhancing%20the%20Josephson%20diode%20effect,of%20the%20Josephson%20diode%20effect), linking topology with nonreciprocal transport—a topic ripe for further theoretical and experimental study.

In conclusion, the Josephson diode effect has evolved from a theoretical curiosity to a vibrant research frontier bridging superconductivity, spin–orbit physics, and device engineering. The theories of finite-momentum pairing and Rashba SOC have provided a solid foundation to understand existing results and predict new phenomena. As materials science advances (e.g. discovery of new non-centrosymmetric or magnetic superconductors) and nanoscale fabrication improves, we anticipate the emergence of superconducting diodes with higher efficiencies, tunable functionalities, and perhaps operating at higher temperatures. Such developments would not only test the limits of the current theories but also potentially lead to technological breakthroughs in ultralow-power superconducting logic and non-dissipative electronic components. Continued dialogue between theory and experiment will be essential to fully unravel the rich behavior of Josephson diode effects and to harness them in practical quantum devices.

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